An efficient identification method of the structural parameters of MDOF structures using the wavelet transform and neural networks

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Abstract

Structures undertaking dynamic load store cumulative damages on their structural members. Although these damages are generally estimated by measuring the acceleration, velocity and displacement at several observing points, the health monitoring system based on these measurements could be expensive.

Therefore, approaches to the health monitoring system utilizing only the acceleration and decreasing the observing points are growing in importance. Hybridizing wavelet analysis and neural networks gives a powerful method for these approaches.

In such a method, wavelet analysis performs efficient time-frequency analysis, which accesses to the information on velocity and displacement from the observed acceleration. On the other hand, neural networks realize a function to estimate the unknown information on points, where the acceleration is not observed, from the data processed by wavelet analysis.

As the first step for this approach, LODE (Linear Ordinary Differential Equation) models with piecewise constant coefficients of MDOF (Multi-Degree Of Freedom) structure will be studied here. These LODE models have stiffness and damping matrices of piecewise constant entries which will be identified from the acceleration. This paper presents an efficient identification method of the structural parameters of MDOF structure using the wavelet transform and neural networks. Some simulations will certify the usefulness of this hybrid method.

1 INTRODUCTION

In this paper, we present a health monitoring system that detect the degradation of structures with Multi-Degree-Of-Freedom (MDOF). As an example of the MODF structures, we consider a building-like structure that is modeled by a linear system shown in Fig.1. This system is composed of three single-degree-of-freedom models with linear stiffness and damping. For notational convenience, the ith material point in Fig.1 is noted as "the ith floor", and the base of the linear system is called "ground floor".

The motion of the structure is simulated by the following Linear Ordinary Differential Equations (LODEs):

$$m_1\ddot{y}_1 + K_1(y_1 - y_2) + C_1(\dot{y}_1 - \dot{y}_2) = 0, \tag{1}$$

$$m_2\ddot{y}_2 + K_1(y_2 - y_1) + C_1(\dot{y}_2 - \dot{y}_1) + K_2(y_2 - y_3) + C_2(\dot{y}_2 - \dot{y}_3) = 0, \tag{2}$$

$$m_3\ddot{y}_3 + K_2(y_3 - y_2) + C_2(\dot{y}_3 - \dot{y}_2) + K_3(y_3 - u) + C_3(\dot{y}_3 - \dot{u}) = 0, \tag{3}$$

where $y_i (i = 1, 2, 3)$ is the displacement of the ith floor, and u is the displacement of the ground floor to which input force is obtained as the acceleration $\ddot{u}$. K_i and C_i are positive stifness and damping paremeters.

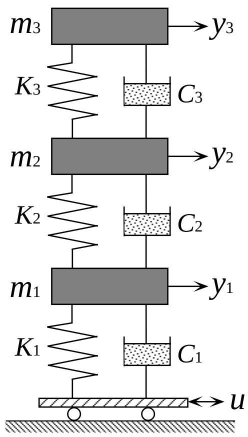

Figure 1: An analytical model of MODF structures.

Let us consider the situation that the dynamical properties of the above MODF structure is gradually changing due to cumulative damages on the structual members which correspond to the degradation in the stifness property of the springs and the damping property of the dampers. In real structures, the parameter changes occur intermittently and the changes are relatively large. Therefore, we can assume that the patermeters K_i and C_i are subjected to a step function of time t. However, since the degradation of real structures is proceeding very slowly as compared with their motion, we can regard the paremters as constants when we observe the motion of the above MODF structure.

If the acceleration $\ddot{y}_i$ at every floor was measured, it is possible to identify all the structural parameters: K_i and C_i. However, in this case, the measurement cost might be too expensive. The purpose of this paper is to present an efficient method for detecting the changes of structural parameters. Therefore, let us consider the following ill-posed problem here: infer all the degradation of structural paremters even when we cannot measure the accelaration at all the floors.

As one of the remedies for this problem, we propose a hibrid method using the wavelet transform and neural networks. In this method, the wavelet analysis performs efficient time-frequency analysis, which accesses to the information on velocity and displacement from the observed acceleration. On the other hand, neural networks realize a function to estimate the unknown information on points, where the acceleration is not observed, from the data processed by wavelet analysis.

In the following sections, we will describe such a hybrid approach to a health monitoring system. Section 2 gives the concrete procedure of applying the wavelet analysis to the health monitoring for MDOF structures. In Section 3, we describe a neural system that the health condition is predicted based on the data processed with the above wavelet analysis. In Section 4, some simulations are carried out in order to evaluate the proposed health monitoring system. Section 5 presents a conclusion.

2 WAVELET TRANSFORM

The acceleration at the third floor and the acceleration at the first floor are observable. It is needed to access to information in frequency space near the natural frequency of the system for identification of Hooke's constants of springs and it is also needed to access to information of energy decay near the natural frequency of the system for identification of damping constants of dampers. For these purposes, wavelet analysis performs efficient time-frequency analysis. Besides time-frequency analysis, wavelets are used as a de-noising tool and as a compression tool. Those observed acceleration data are de-noised and compressed.

When the positions of all the floors are observable, [1] expanded those positions into the wavelet expansion by orthonormal wavelet bases and identified Hooke's constants and damping constants using the least square method. In this paper, since the observable are only acceleration at two points, neural networks are used to estimate structural parameters. For simplicity, only a model of MDOF structure having three floors will be studied. This model has six structural parameters. Each floor has two parameters, one is Hooke's constant and the other is the damping constant. We will give a simulation to identify those structural parameters from acceleration at the third floor and at the first floor. The acceleration are processed by wavelet analysis first and identified by neural networks. We will explain the processing method by wavelet analysis in this section.

For the wavelet function $\psi(t)$, the *continuous wavelet transform* of $f(t)$ is defined by

$$(W_\psi f)(a, b) = \frac{1}{\sqrt{a}} \int f(t) \psi\left(\frac{t-b}{a}\right) dt. \tag{4}$$

Here a is called the *dilation parameter* and b is called the *position parameter*.

The continuous wavelet analysis is often easier to interpret than the discrete wavelet analysis, such as the wavelet expansion with a wavelet orthonormal basis, since its redundancy tends to reinforce the traits and makes all information more visible.

We discretize the dilation parameter and the position parameter in (4) carefully. We want to optimize discretization in dilation, which corresponds to frequency, and to have redundancy in position, which corresponds to time.

We employ Meyer's orthonormal wavelet $\psi(t)$ such that, for $N \in \mathbf{N}$ and $\alpha = (N+1)/N$, the system

$$\left\{\alpha^{j/2} \psi\left(\alpha^j t - k\right)\right\}_{j,k \in \mathbf{Z}} \tag{5}$$

is an orthonormal basis. Here j is called the *scale parameter*. Notice that the larger N becomes, the finer the discretization in frequency becomes.

Choose $N = 16$ and $\alpha = (N+1)/N = 17/16 = 1.0625$. The graph of $\psi(t)$ for $\alpha = 17/16$ is shown in Fig. 2. We discretize the dilation parameter a by $a = \alpha^{-j}$, $j \in \mathbf{Z}$. Since the discretization for the case of an orthonormal wavelet basis is the minimal discretization, this choice means that we minimize the discretization in frequency for given N.

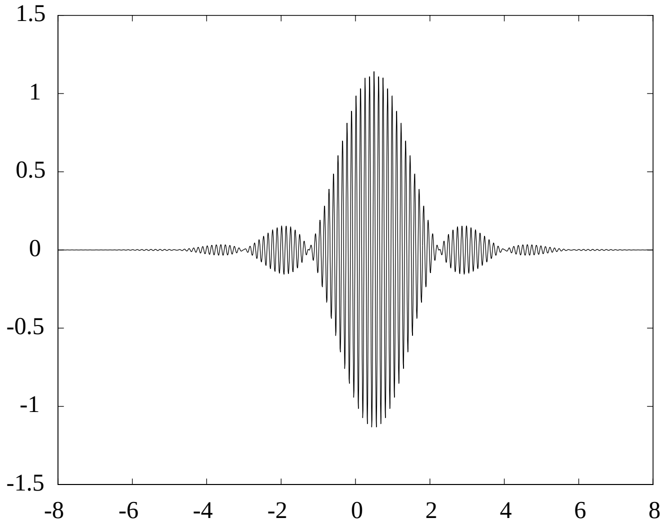

Figure 2: Meyer's wavelet ψ for $\alpha = 17/16$.

Since

$$\operatorname{supp} \widehat{\psi}(\omega) = \left[-\frac{2(N+1)^2\pi}{2N+1}, -\frac{2N^2\pi}{2N+1} \right] \cup \left[\frac{2N^2\pi}{2N+1}, \frac{2(N+1)^2\pi}{2N+1} \right],$$

the central frequency of ψ is $\frac{N(N+1)}{2N+1} = \frac{16\times17}{33} = 8.24$ Hz and the central frequency of $\alpha^{j/2}\psi\left(\alpha^j t - k\right)$ is $8.24\alpha^j$ Hz.

Since the natural frequency of the system is approximately 1 Hz, we use eight wavelets from the scale $j = -38$, which corresponds to the central frequency $\alpha^j = 0.82$ Hz, to the scale $j = -31$, which corresponds to the central frequency 1.26 Hz.

As for the discretization of the position parameter b, since the step size must be no more than $1/\alpha^j$, which corresponds to the orthonormal case of (5), it is enough to choose $1/\alpha^j = 10.0$ for $j = -38$ and 6.55 for $j = -31$. For redundancy and for simplicity in calculation, we discretize b with the same step size, which is 1 sec, at every scale. We need to pay an attention to repetition caused by this redundancy, when we calculate the L^2-energy of data. The L^2-energy of data corresponding to the scale j has approximately α times repetition comparing to the L^2-energy of data corresponding to the scale $j+1$.

The wavelet coefficient C_k^j of the acceleration data $f(t)$ for the scale j and the position k sec is given by

$$C_k^j = \int f(t)\alpha^{j/2}\psi\left(\alpha^j(t-k)\right)\,dt. \tag{6}$$

The L^2-energy of the frequency band corresponding to the scale j is estimated by

$$E^j = \alpha^j\sqrt{\sum\nolimits_k |C_k^j|^2}. \tag{7}$$

The L^2-energy near the time k is estimated by

$$E_k = \sqrt{\sum_j |\alpha^j C_k^j|^2}. \tag{8}$$

Denote by C_k^j the wavelet coefficients of the acceleration at the third floor and by D_k^j the wavelet coefficients of the acceleration at the first floor.

3 Neural Networks

To predict the deterioration of stiffness and damping in MODF structures, we utilize conventional three-layer neural networks (NNs) whose information processing is given by

$$o_i^{(l+1)} = f(\sum_j w_{ij}^{(l)} o_j^{(l)} + \theta_i^{(l+1)}) \quad (l = 1, 2), \tag{9}$$

where

$$f(x) = \frac{1}{1 + \exp(-x)}. \tag{10}$$

Here, $o_i^{(l)}$ is the output of the ith unit at the lth layer, i.e. $o_i^{(1)}$ and $o_i^{(3)}$ correspond to the input and the final output of the NN. $\theta_i^{(l)}$ is the ith threshhold at the lth layer, and $w_{ij}^{(l)}$ means a connection weight from the jth unit at the lth layer to the ith unit at the $(l+1)$th layer.

Let us consider the case of the building-like structure stated in Section 1. In this case, the inputs $o_i^{(1)}$ correspond to the information on the observed acceleration at the first and fourth floor. This network inputs are generated from the observed acceleration after a certain normalization (see Section 4):

As described in Section 1, the health condition of a MODF structure is reflected to the estimation of the structural parameters in the LODEs model (i.e. stiffness and damping). We construct the NN such that the outputs represent the estimation for a structural parameter; thus, we prepare six 3-layered NNs for the estimation of $K_1 \sim K_3$ and $C_1 \sim C_3$. For each structural parameter, three levels of health conditions are considered: normal, 10% and 20% degredation. The confidence for the health conditions are respectively represented by the three outputs $o_1^{(3)}$, $o_2^{(3)}$ and $o_3^{(3)}$. The correspondence between the health conditions and network outputs is defined as shown in Table 1. For example, when $o_2^{(3)}$ is the largest, we can see that the NN predicts the 10% degradation of the corresponding parameter (stiffness/damping).

In the learning phase, we adopt the extended Back-Propagation method in which the one dimensional search and the forgetting effect are introduced at every weight modification.

4 Simulations

The settings of structual parameters and the external force are defined as follows. Let the masses be $m_1 = m_2 = m_3 = 1.0$, Hooke's constants be $K_1 = K_2 = K_3 = 240.0$, and the damping constants be $C_1 = C_2 = C_3 = 1.55$ for normal values. Then, the natural frequency of the system is approximately 1 Hz. We assume that these parameters are step functions. Assume that each Hooke's constant K_1, K_2, K_3 has three different values 240.0 (normal value), 216.0 (90% of the normal value), 192.0 (80% of the normal

Table 1: Target outputs of the NN according to the health conditions.

health condition	z_1	z_2	z_3
normal	0.8	0.2	0.2
10% degradation	0.2	0.8	0.2
20% degradation	0.2	0.2	0.8

value) and that each damping constant C_1, C_2, C_3 has also three different values 1.55 (normal value), 1.705 (110% of the normal value), 1.86 (120% of the normal value). Hence, the number of cases to be studied are $3^6 = 729$. The MDOF structure is vibrated by the external force. For the acceleration of the external force, we input normal random number made by random seed = 5. Assume that the MDOF structure begins to vibrate at the time $t = 0$. The ordinary differential equation for the MDOF structure is solved by the forth order Runge–Kutta method with the step size 1/1000 sec. We will use the wavelet transform of the data from 200 sec to 1200 sec.

We define the following notaton. U_n^j is the L^2 average of j-th frequency scale wavelet coefficients for 20 sec, from n sec, at the third floor of the MDOF structure. And V_n^j denotes the same average at the first floor.

$$U_n^j = \frac{\alpha^j}{20}\sqrt{\sum_{k=n}^{n+19} |C_k^j|^2}, \qquad V_n^j = \frac{\alpha^j}{20}\sqrt{\sum_{k=n}^{n+19} |D_k^j|^2}.$$

$V_n = \sqrt{\sum_{j=-38}^{-31} |V_n^j|^2}$ denotes the total average energy at the first floor, and $U_n = \sqrt{\sum_j |U_n^j|^2}$ is the total average energy at the third floor. This energy V_n is used for the normalizing factor of inputs of the neural network.

We use three-layer neural network model which consists of 45 inputs , 50 hidden units and 3 outputs. These 45 inputs are composed of 5 sets of 9 components. These 9 components are calculated from wavelet coefficients at the same time location. At n sec, The first components is U_n/V_n. And the second to 5th components are $U_n^{-36}/V_n, U_n^{-35}/V_n, U_n^{-34}/V_n$,and U_n^{-33}/V_n ,respectively. The following 6th to 9th components are $V_n^{-36}/V_n, V_n^{-35}/V_n, V_n^{-34}/V_n$,and V_n^{-33}/V_n ,respectively. Then our 9 components made into a bandle I_n. Here

$$I_n = \left(\frac{U_n}{V_n}, \frac{U_n^{-36}}{V_n}, \frac{U_n^{-35}}{V_n}, \frac{U_n^{-34}}{V_n}, \frac{U_n^{-33}}{V_n}, \frac{V_n^{-36}}{V_n}, \frac{V_n^{-35}}{V_n}, \frac{V_n^{-34}}{V_n}, \frac{V_n^{-33}}{V_n}\right).$$

Finarry, the 45 inputs of this neural network consist of I_n, I_{n+10}, I_{n+20}, I_{n+30}, and I_{n+40}.

We make the training data as follows. We set random seed = 5, and the MDOF structure is vibrated by the external force. For fixed Hooke's constants and damping constants, while this external force is applied to our MDOF structure, wavelet coefficients of the acceleration at the third and first floors are calculated until 1200 sec. From 200 sec, the training data are made every 100 sec. Then 10 training data are gotten for one set of Hooke's and damping constants. Next, Hooke's and damping constans change to another set, and the training data are made. Because 3 Hooke's constants

Hooke's constant	K_3		K_2		K_1	
Learning of Hooke's constant 7290 data						
First Total Error	1723.7	929.2	1477.9	1217.3	963.2	1526.2
Last Total Error	308.9	434.1	243.4	173.6	153.1	98.6
Percentage of Correct Answers	95	90	98	100	100	100
Prediction of Hooke's constant 14980 data						
Total Error	914.2	1275.6	786.2	781.5	469.3	391.7
Percentage of Correct Answers	80	70	86	87	94	97

Table 2: Learning and prediction of Hooke's constant $K1, K2, K3$

and the 3 damping constants have 3 value respectively, $3^6 = 729$ sets of constants are produced. Therefore, we get 7290 training data.

For making the test data, the random seed = 7 is set, and the same procedure is carried out. From 200 sec, this test data are made every 50 sec. In this case, 20 test data are made for one set of Hooke's and damping constants. Finally, we get 14980 test data.

For 3 Hooke's constans K_1, K_2, and K_3 respectively, training neural networks and predicting the test data are carried out. The results are in Table 2. If the state of maximum of outputs $o_1^{(3)}, o_2^{(3)}, o_3^{(3)}$ agrees with the spring health condition at that time, then this set of outputs is called a correct answer. The neural network is trained 2 times for each spring. About Hooke's constants, training and prediction are also possible.

For damping constants C_1, C_2, C_3, training neural network and predicting the test data also are carried out. However, this procedure did not go well. For exmaple, in the case of C_1, the first total error of this neural network is 1184.8, and the last total is 874.1. The percentage of correct answers is 34%. The total error of test data is 2730.5, and percentage of correct answers is 35%.

5 Conclusion

For K_1 ,it is easier to train neural network and to predict than for K_2 or K_3. For K_1, the correlation between 45 inputs and 3 target outputs is stronger than for K_2 or K_3. That the same external force is adding to the MDOF structure may be a cause. Concerned with the health condition of damping constants, this procedure did not go well.

This MDOF structure model is a system of second order linear ordinary differential equations. However Hooke's constants can be determined from only acceleration, since Hooke's constants are the coefficients of the 0 order derivatives (displacement of the structure). By the way, the damping constants are the coefficients of the 1 order derivatives (velocity of the structure). Therefore, it is impossible to estimate the damping constants from only acceleration. In addition to acceleration, the velocity and displacement of the structure will be required.

To make the neural network for an estimate of only one damping constant C_1 or C_2 or C_3, the wavelet coefficients of velocity and displacement are calculated form the acceleration by the following. $f'(t)$ denotes the acceleration, and the wavelet coefficients of velocity $f(t)$ are calculated as a convolution between acceleration and derivative of wavelet $\psi'(t)$ by integration by parts. The wavelet coefficients of $f''(t)$ are also calculated as a convolution between acceleration and integral of wavelet $\Psi(t) = \int_{1/2}^{t} \psi(x)dx$.

$$(W_\psi f)(a,b) = \frac{1}{\sqrt{a}} \int f(t)\psi\left(\frac{t-b}{a}\right) dt = -\sqrt{a} \int f'(t)\Psi\left(\frac{t-b}{a}\right) dt \tag{11}$$

$$(W_\psi f'')(a,b) = \frac{1}{\sqrt{a}} \int f''(t)\psi\left(\frac{t-b}{a}\right) dt = -\frac{1}{a\sqrt{a}} \int f'(t)\psi'\left(\frac{t-b}{a}\right) dt \tag{12}$$

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