



2019 RIMS 共同研究 「多次元 Stockwell 変換と時間周波数解析」

数学アドバンスイノベーションプラットフォーム (AIMaP)

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会場：RIMS ANNEX 数理解析研究所別館 (総合研究 15 号館：旧建築学教室本館) 201 号室

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## プログラム

11 月 6 日 (水)

13:00 – 14:00 **Luigi Riba** (Data engineer and product specialist, Irion, Italy)

### **A link among multi-dimensional Stockwell transform, Gabor transform and wavelet transform**

The reasoning behind the definition of the multi-dimensional Stockwell transform is described in detail. Several continuous inversion formulas for the multi-dimensional Stockwell transform under different sets of hypothesis are provided.

14:15 – 15:15 **Tamotu Kinoshita** (Mathematics, University of Tsukuba)

### **On directional frames having Lipschitz continuous Fourier transforms**

As a discretization of two-dimensional wavelet transforms, we study directional frames having Lipschitz continuous Fourier transforms based on concentric regular  $2^N$ -sided polygons in the frequency domain. Especially, we consider a new simple way of expansions with non-Parseval frames for better reconstructions. Moreover, we also propose a discretization scheme of the inversion of the Radon transform, and represent the solution of the wave equation.

15:45 – 16:45 **Kazuaki Nakane** (Internal Medicine, Osaka University)

### **Cytology support system via topological idea**

Cytodiagnosis is a method for diagnosing whether cancer cells are contained in cells collected from the human body. Because it can be performed easily, the number of diagnoses is increasing year by year. The shortage of pathologists is serious problem in global, and the development of assistive technology using computers is an urgent issue. In this talk, the method using topological ideas and several results are introduced.

11月7日(木)

9:30 – 10:30 **Luigi Riba** (Data engineer and product specialist, Irion, Italy)

**Discrete orthonormal Stockwell transform and two-dimensional applications**

The discrete orthonormal Stockwell transform, its properties and its extension to the multi-dimensional case are described. An application of Stockwell transform to Ground Penetrating Radar (GPR) images is given.

10:45 – 11:45 **Shinya Moritoh** (Mathematics, Nara Women's University)

**Two-microlocal estimates in wavelet theory and related function spaces**

We talk about two-microlocal estimates in wavelet theory and related function spaces, which is a continuation of our work in 2004 and 2016. Some applications are also considered.

13:15 – 14:15 **Kensuke Fujinoki** (Mathematical Sciences, Tokai University)

**Aspects of frame analysis**

Frame theory has been known as a branch of functional analysis while a frame itself has also been known as an indispensable tool used in a wide range of data analysis, especially signal processing. In this talk we overview the basic and general theory for frame analysis, and introduce the recent developments and applications in signal processing including sampling theory, Gabor systems and wavelet systems.

14:30 – 15:30 **Zhong Zhang** (Mechanical Engineering, Toyohashi University of Technology)

**Cracks in Concrete Detected by 2 Dimensional Complex Discrete Wavelet Packet Transform using complex-valued Haar wavelet**

In this study, we focused on the ability to detect cracks, and designed a complex-valued Haar wavelet, and applied it to 2D-CWPT. 2D-CWPT and anisotropic diffusion filter were combined, and tested, and a crack detection method for linear cracks was proposed through interpolation of the crack loss area and extraction processing according to the shape feature.

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# RIMS Workshop 2019

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## A link among multi-dimensional Stockwell transform, Gabor transform and Wavelet transform

Stockwell Transform (S-Transform) has been introduced in [1] (1996) by R.G. Stockwell, L. Mansinha and R.P. Lowe. During the years this transform has found application in medical imaging, geophysics and signal processing in general.

In [2] (2006) M. W. Wong and H. Zhu studied from a theoretical perspective the 1-dimensional Stockwell transform defined as:

$$S_{\varphi} f(b, \xi) = (2\pi)^{-1/2} |\xi| \int_{\mathbb{R}} e^{-ix\xi} f(x) \overline{\varphi(\xi(x-b))} dx$$

In [3] (2007) Y. Liu and M. W. Wong introduced multiple versions of two-dimensional Stockwell transform proving inversion formulas for them. In 2013 this work has been extended by L. Riba and M. W. Wong who defined a multi-dimensional Stockwell transform as:

$$S_{A, \varphi} f(b, \xi) = (2\pi)^{-n/2} |\det A_{\xi}|^{-1} \int_{\mathbb{R}^n} e^{-ix \cdot \xi} f(x) \overline{\varphi(A_{\xi}^{-1}(x-b))} dx$$

where  $A : \mathbb{R}^n \ni \xi \rightarrow A(\xi) = A_{\xi} \in GL(n, \mathbb{R})$  .

This multi-dimensional Stockwell transform is linked to the Affine-Heisenberg group. This connection shows the deep link that exists among the Stockwell transform, the Gabor transform and the Wavelet transform.

In this talk, following the work done in [4] (2014), we aim to provide a theoretical setting in which to study the Stockwell transform. In particular, we are going to describe in detail the reasoning behind the definition of the multi-dimensional Stockwell transform. Furthermore, we are going to provide several continuous inversion formulas for the multi-dimensional Stockwell transform under different sets of hypothesis on the matrix valued function  $A_{\xi}$  . In the end, we plan to show the relation between the phase of the multi-dimensional Stockwell transform and the instantaneous frequency of a signal.

## References

- [1] R. G. Stockwell, L. Mansinha, and R. P. Lowe., 1996. Localization of the complex spectrum: the S-transform., IEEE Transactions on Signal Processing , 44: 998-1001.
- [2] M. W. Wong and H. Zhu, A characterization of the Stockwell spectrum, in Modern Trends in Pseudo-Differential Operators, Birkhauser, 2007, 251–257.
- [3] Y. Liu, M. W. Wong, Inversion formulas for two-dimensional Stockwell transforms, Pseudo-differential Operators: Partial Differential Equations and Time-Frequency Analysis, Fields Institute Communications, 52 (2007), pp. 323–330.
- [4] L. Riba, Multi-Dimensional Stockwell Transforms and Applications, PhD thesis, Università degli Studi di Torino, Italy, 2014.

# Discrete Orthonormal Stockwell Transform and two-dimensional applications

The Stockwell Transform (S-Transform) has been defined in [1] (1996) by R.G. Stockwell, L. Mansinha and R.P. Lowe. Due to its redundant nature and its computational complexity (  $O(N^2 \log N)$  ) its range of applicability has been initially limited to moderately sized signals.

In [2] (2007) R. G. Stockwell himself overcame the redundancy introducing an orthogonal basis linked to the S-Transform on which it is possible to decompose a signal via the so called Discrete Orthonormal Stockwell Transform (DOST). The same paper exhibits an algorithm to compute the DOST coefficients with a, better but still rather high, computational complexity of (  $O(N^2)$  ). This paper led Y. Wang and J. Orchard to find in [3] (2009) a FFT-fast (  $O(N \log N)$  ) way to compute the DOST coefficients. This work broadened the range of applications of the transform.

In [4] (2016) U. Battisti and L. Riba gave a rigorous proof of the orthonormality of the so called DOST basis and shown a (  $O(N \log N)$  ) algorithm to compute Stockwell transform which extends the one proposed by Y. Wang and J. Orchard.

In this talk, following the work done in [4], we are going to discuss in details the Discrete Orthonormal Stockwell Transform, its properties and its extension to the multi-dimensional case. In particular, we are going to present the work done in [5] (2014) by L.Riba, S. Piro, U. Battisti and L. Sambuelli in 2015 about the application of Stockwell transform to Ground Penetrating Radar (GPR) images.

## References

- [1] R. G. Stockwell, L. Mansinha, and R. P. Lowe., 1996. Localization of the complex spectrum: the S-transform., IEEE Transactions on Signal Processing , 44: 998-1001.
- [2] R. G. Stockwell, "A basis for efficient representation of the S-transform," Digital Signal Processing, vol. 17, no.1, pp. 371–393, January 2007
- [3] Y. Wang, J. Orchard, Fast discrete orthonormal Stockwell transform, SIAM J. Sci. Comput. 31 (2009) 4000–4012.
- [4] U. Battisti, L. Riba, Window-Dependent Bases for Efficient Representations of the Stockwell Transform, Applied and Computational Harmonic Analysis, Vol. 40, Issue 2, pp. 292-320, February 2015
- [5] L. Riba, S. Piro, U. Battisti, L. Sambuelli, On the application of the Stockwell transform to GPR data analysis, Advanced Ground Penetrating Radar (IWAGPR), p. 1- 4