

Characterization of a uniform domain by the boundary Harnack principle

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Ever since the pioneering works of Carleson [8] and Hunt-Wheeden [10, 11] for Lipschitz domains, a large number of works have been devoted to the study of potential theory for non-smooth domains, such as Lipschitz domains, NTA domains, uniform domains, John domains and Hölder domains. Among them, the boundary Harnack principle and the identification of the Martin boundary played a central role. See Ancona [5], Dahlberg [9] and Wu [13] for Lipschitz domains, Jerison-Kenig [12] for NTA domains, Bass-Burdzy [7] for Hölder domains, HA [1] for uniform domains, Balogh-Volberg [6] and ALM [4] for uniformly John domains. See [2] for a survey. These studies are motivated by the desire to generalize the geometric conditions imposed on a domain to guarantee potential theoretic properties, such as the boundary Harnack principle.

This talk is oriented in the opposite direction. Namely, we shall give some potential theoretic properties which yield geometric properties of the domain. Our conditions will be necessary and sufficient, provided the domain satisfies the capacity density condition for the complement. We shall introduce several non-smooth domains such as uniform domains, John domains, uniformly John domains, inner uniform domains and so on. Under the capacity condition, we shall characterize these domains by a certain estimate of harmonic measure and the boundary Harnack principle. The ingredient of this talk is published in [3].

References

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