運動の相対性について

伊東由文　(徳島大学名誉教授)

本論文においては, アインシュタインの相対性理論の意味について考察する.

絶対空間と物体の運動　物理現象としての物体の運動は人間による観測とは無関係に現象している. このとき, 物体の運動している空間は座標系のとり方には無関係な存在である. そういう意味で物理空間は「絶対空間」である. そのような物理空間における物体の運動を研究するためには, 一つの座標系を固定して物体の位置ベクトルの時間的変動として物体の運動を表現する. ニュートンは, 慣性系を考えた上で, 力学の法則がどう表されるかを調べている.

空間と時間　物体の運動を考えるとき, 空間の位置と時間についての明確な規定が必要である. このとき, 空間と時間の概念については著しい相異点があることに注目する必要がある.

実際に実在の物理空間は 3 次元ユークリッド空間であることが実証されている. それ故に, 物体の運動について考察するとき, 理論モデルにおいて, 空間は数学概念としての 3 次元ユークリッド空間であると考える. これと独立に 1 次元の時間軸を考える.

このとき, 3 次元ユークリッド空間において xyz 軸が互いに直交するということはいえるが, 時間軸がこれらと直交するということは意味がない.

さらに, 空間においては幾何学的な形を考えることができるが, 時間軸に関しては, このような形を考えることはできない. 特に, 空間においては真っ直ぐな直線を考えることができるが, 時間軸が真っ直ぐな形をしているということを考えることはできない.

3 次元ユークリッド空間において, 点の位置座標は一つの直交座標系を定めることによって決められる.

ニュートンの運動法則は, 理論モデルに対して表現されているから, 当然考える座標系に依存している. 現実の物理空間が 3 次元ユークリッド空間であることから, 物体の運動の観測に用いることのできる座標系は直交座標系に限られる. それ故に, 二つの直交座標系が相対運動できるのは原点の移動と直交座標軸の回転運動に限られる. したがって, 全く自由に座標系を変えられるわけではない. このことから考えると, ニュートンの運動方程式に従う物体の運動の相対性は二つの直交座標系の相対運動による見かけの運動について考えることになる.

物体の運動に対応する理論モデルは, 数学的概念としての 3 次元ユークリッド空間において, 一つの直交座標系を定めて, 質点として表された点の位置ベクトルの時間に依存する変動として表される. その運動法則はニュートンの運動方程式として定立される.

質点の位置座標は質点の運動と共にニュートンの運動方程式に従って変動する. それに対し, 時間は質点の運動とは独立に変動し, ニュートンの運動方程式に従って変動する量ではない.

現象世界において観測に用いる座標系のとり方を変えることは, 理論モデルとしては, 二つの座標系の間の座標変換として表される. このとき, 二つの座標系は等速度で相対運

動をしているか, 加速度によって相対運動をしているかの二つの場合が考えられる. 理論的には, 同一の時刻において空間の点の位置座標を二つの座標系に対して同時に決めることができる.

運動の相対性　物体の運動を観測するとき, 原理的には直交座標系のとり方は任意である. しかし, 二つの座標系の相対運動の様相に応じて, 二つの座標系による物体の運動の見かけの様相は一般に異なっている. このような座標系によって見かけの運動は違っていることを理解する理論が相対性理論である.

「慣性の法則が成り立つ座標系を慣性座標系, または慣性系である」という表現は正確ではない.

一つの座標系を決めてニュートンの運動方程式を表現するのであるから, 座標系のとり方とニュートンの運動の三法則の表現は互いに無関係ではあり得ない. ニュートンの運動方程式において力の項が 0 になるような座標系が「慣性系」ということである.

慣性系であるかどうかは考える現象の運動方程式との関連によって決められるべき概念である.

したがって, ある座標系をとって, ニュートンの運動法則を表現するとき, 見かけの運動として, 慣性の法則によって運動しているかどうかということが考えられるのである. それ故に, 「慣性の法則」が成り立つということも考えている座標系に依存することである.

一つの座標系において慣性の法則が成り立っているとき, それに対し, 等速度相対運動をしている他の座標系を用いて見たときにもやはり慣性の法則が成り立っている.

それ故に, ある座標系が「慣性系」であるということは, ニュートンの運動方程式に従って運動する物体の見かけの運動が慣性の法則に従って運動していることであると定義することができる. ニュートンの運動方程式を用いないで物体の運動が等速直線運動をしているということは直観的な考え方である. 物体の運動についての考察は, 物体の運動法則であるニュートンの運動方程式に基づいて議論しなければならないから, ニュートンの運動方程式と無関係に慣性系を考えることは意味がない. 物体に力が作用していないときにも, 二つの座標系の一つは慣性系であるとして, もう一つの座標系がその慣性系に対して加速度運動をしていれば, そのような座標系に対してはニュートンの運動方程式は力の作用の因子が加わることになって, その座標系は慣性系にはならないということである. それ故に, 同じ物体の運動に対してであっても座標系のとり方によってニュートンの運動方程式の形がかわる. したがって, 物体の見かけの運動が変わる. すなわち, 物体の運動の軌道を表す曲線の形が変わるのである. それにもかかわらず, その物体の運動そのものは, 見かけの運動の形の違いにもかかわらず同一の物体の同一の運動である. このことを数理モデルを用いて考える理論が相対性理論である.

さらに, ニュートンの運動方程式には光速 c に依存する因子はないから, 質点の運動は光速 c には無関係である. さらに, ローレンツ変換に現れる直交座標と時間は直接光速 c に依存しているわけではない. それにもかかわらず, どうして特殊相対性理論においては光速 c が質点の運動に影響するのか不明である.

詳細については講演のときに考察する.

(2014.10.25)

WEIGHTED NORM INEQUALITIES FOR MULTILINEAR FOURIER MULTIPLIERS WITH CRITICAL BESOV REGULARITY

MAI FUJITA (WAKKANAI HOKUSEI GAKUEN UNIVERSITY)

In this talk, we consider weighted norm inequalities for multilinear Fourier multipliers with critical Besov regularity. As a result, we obtain a limiting case of Hörmander type multiplier theorem for multilinear operators. This talk is based on a joint work with Naohito Tomita (Osaka University).

For $m \in L^\infty(\mathbb{R}^{Nn})$, the N-linear Fourier multiplier operator T_m is defined by

$$T_m(f_1, \dots, f_N)(x) = \frac{1}{(2\pi)^{Nn}} \int_{(\mathbb{R}^n)^N} e^{ix\cdot(\xi_1+\cdots+\xi_N)} m(\xi) \widehat{f_1}(\xi_1) \dots \widehat{f_N}(\xi_N)\, d\xi$$

for $f_1, \dots, f_N \in \mathcal{S}(\mathbb{R}^n)$, where $x \in \mathbb{R}^n$, $\xi = (\xi_1, \dots, \xi_N) \in (\mathbb{R}^n)^N$ and $d\xi = d\xi_1 \dots d\xi_N$. Let $\Psi \in \mathcal{S}(\mathbb{R}^{Nn})$ be such that

$$\operatorname{supp} \Psi \subset \left\{\xi \in \mathbb{R}^{Nn} : 1/2 \le |\xi| \le 2\right\}, \quad \sum_{k\in\mathbb{Z}} \Psi(\xi/2^k) = 1, \quad \xi \in \mathbb{R}^{Nn} \setminus \{0\},$$

and set

$$m_j(\xi_1, \dots, \xi_N) = m(2^j\xi_1, \dots, 2^j\xi_N)\Psi(\xi_1, \dots, \xi_N), \quad j \in \mathbb{Z}.$$

We denote by $\|T_m\|_{L^{p_1}(w_1)\times\cdots\times L^{p_N}(w_N)\to L^p(w)}$ the smallest constant C satisfying

$$\|T_m(f_1, \dots, f_N)\|_{L^p(w)} \le C \prod_{i=1}^N \|f_i\|_{L^{p_i}(w_i)}, \quad f_1, \dots, f_N \in \mathcal{S}(\mathbb{R}^n),$$

where the weighted Lebesgue space $L^p(w)$ for $0 < p < \infty$ and $w \ge 0$ consists of all measurable functions f on $\mathbb{R}^n$ such that

$$\|f\|_{L^p(w)} = \left(\int_{\mathbb{R}^n} |f(x)|^p w(x)\, dx\right)^{1/p} < \infty.$$

In the unweighted case, Tomita [3] proved a Hörmander type multiplier theorem for multilinear operators, namely, if $s > Nn/2$ then

$$\|T_m\|_{L^{p_1}(\mathbb{R}^n)\times\cdots\times L^{p_N}(\mathbb{R}^n)\to L^p(\mathbb{R}^n)} \lesssim \sup_{j\in\mathbb{Z}} \|m_j\|_{H^s(\mathbb{R}^{Nn})} \tag{1}$$

for $1 < p_1, \dots, p_N, p < \infty$ satisfying $1/p_1 + \cdots + 1/p_N = 1/p$, where $H^s(\mathbb{R}^{Nn})$ is the Sobolev space of usual type. Grafakos-Si [2] extended this result to the case $p \le 1$ by using the L^r-based Sobolev spaces, $1 < r \le 2$. Let $1 < p_1, \dots, p_N < \infty$ and $1/p_1 + \cdots + 1/p_N = 1/p$. In the weighted case, Fujita-Tomita [1] proved that if $n/2 < s_j \le n$, $p_j > n/s_j$ and $w_j \in A_{p_j s_j/n}$, $1 \le j \le N$, then

$$\|T_m\|_{L^{p_1}(w_1)\times\cdots\times L^{p_N}(w_N)\to L^p(w)} \lesssim \sup_{j\in\mathbb{Z}} \|m_j\|_{H^{(s_1,\dots,s_N)}((\mathbb{R}^n)^N)}, \tag{2}$$

where $w = w_1^{p/p_1} \dots w_N^{p/p_N}$ and $H^{(s_1,\dots,s_N)}((\mathbb{R}^n)^N)$ is the Sobolev space of product type.

Our purpose in this talk is to consider the limiting cases $s = Nn/2$ in (1) and $s_j = n/2$, $1 \le j \le N$ in (2).

References

[1] M. Fujita and N. Tomita, Weighted norm inequalities for multilinear Fourier multipliers, Trans. Amer. Math. Soc. 364 (2012), 6335-6353.

[2] L. Grafakos and Z. Si, The Hörmander multiplier theorem for multilinear operators, J. Reine Angew. Math. 668 (2012), 133-147.

[3] N. Tomita, A Hörmander type multiplier theorem for multilinear operator, J. Funct. Anal. 259 (2010), 2028–2044.

Mai Fujita, Faculty of Integrated Media, Wakkanai Hokusei Gakuen University, Wakkanai, Hokkaido 097-0013, Japan

E-mail address: fujita@wakhok.ac.jp

Characterization of generalized Besov Morrey spaces by ball means of differences

Takahiro Noi (Tokyo Metropolitan University)
1-1 Minami-osawa, hachioji-city, Tokyo
E-mail : taka.noi.hiro@gmail.com

1 Definition of function spaces

Let $\mathcal{Q}$ denote the set of all compact cubes whose edges are parallel to the coordinate axes. Given a cube Q, we denote by $\ell(Q)$ the *side-length of* Q: $\ell(Q) = |Q|^{1/n}$, where $|Q|$ denotes the volume of the cube Q.

Let $0 < q < \infty$. Denote by $\mathcal{G}_q$ the set of all nondecreasing functions $\varphi : (0, \infty) \to (0, \infty)$ such that

$$\varphi(t_1){t_1}^{-n/q} \geq \varphi(t_2){t_2}^{-n/q} \quad (0 < t_1 \leq t_2 < \infty). \tag{1}$$

Definition 1.1 (Generalized Morrey spaces [1])**.** Let $0 < q < \infty$ and $\varphi \in \mathcal{G}_q$. Then define

$$\|f\|_{\mathcal{M}_q^\varphi} \equiv \sup_{Q \in \mathcal{Q}} \varphi(\ell(Q)) \left(\frac{1}{|Q|} \int_Q |f(y)|^q \, dy \right)^{\frac{1}{q}}$$

for a measurable function f. The space $\mathcal{M}_q^\varphi(\mathbb{R}^n)$ is the set of all measurable functions f for which the quasi-norm $\|f\|_{\mathcal{M}_q^\varphi}$ is finite.

If a cube Q has center 0 and radius r, we denote it by $Q(r)$. That is,

$$Q(r) = \left\{ y \in \mathbb{R}^n \, : \, \max_{1 \leq i \leq n} |y_i| \leq r \right\}.$$

In a standard way, we extend the definition of $\mathcal{F}$ and $\mathcal{F}^{-1}$ to the space of all tempered distributions $\mathcal{S}'(\mathbb{R}^n)$.

For $\varphi \in \mathcal{S}(\mathbb{R}^n)$ and $f \in \mathcal{S}'(\mathbb{R}^n)$ we write $\varphi(D)f \equiv \mathcal{F}^{-1}[\varphi \mathcal{F} f]$, or equivalently we define

$$\varphi(D)f(x) \equiv \frac{1}{\sqrt{(2\pi)^n}} \langle f, \mathcal{F}^{-1}\varphi(x - \cdot) \rangle.$$

Definition 1.2 (Generalized Besov Morrey spaces [2])**.** Let $0 < q < \infty$, $0 < r \leq \infty$, $s \in \mathbb{R}$ and $\varphi \in \mathcal{G}_q$. Let θ and τ be compactly supported functions satisfying

$$0 \notin \mathrm{supp}(\tau), \quad \theta(\xi) > 0 \text{ if } \xi \in Q(2), \quad \tau(\xi) > 0 \text{ if } \xi \in Q(2) \setminus Q(1).$$

Define $\tau_k(\xi) \equiv \tau(2^{-k}\xi)$ for $\xi \in \mathbb{R}^n$ and $k \in \mathbb{N}$.

The (nonhomogeneous) generalized Besov-Morrey space $\mathcal{N}^s_{\mathcal{M}^\varphi_q,r}(\mathbb{R}^n)$ is the set of all $f \in \mathcal{S}'(\mathbb{R}^n)$ for which the quasi-norm

$$\|f\|_{\mathcal{N}^s_{\mathcal{M}^\varphi_q,r}} \equiv \begin{cases} \|\theta(D)f\|_{\mathcal{M}^\varphi_q} + \left(\sum_{j=1}^{\infty} 2^{jsr}\|\tau_j(D)f\|_{\mathcal{M}^\varphi_q}{}^r\right)^{\frac{1}{r}} & (r<\infty), \\ \|\theta(D)f\|_{\mathcal{M}^\varphi_q} + \sup_{j\in\mathbb{N}} 2^{js}\|\tau_j(D)f\|_{\mathcal{M}^\varphi_q} & (r=\infty) \end{cases}$$

is finite.

2 Characterization by ball means of differences

In this talk, we consider the characterization of $\mathcal{N}^s_{\mathcal{M}^\varphi_q,r}(\mathbb{R}^n)$ by ball means of differences.

Let f be a function on $\mathbb{R}^n$ and let $h \in \mathbb{R}^n$. Then we define

$$\triangle^1_h f(x) = f(x+h) - f(x), \quad x \in \mathbb{R}^n.$$

The higher order differences are defined inductively by

$$\triangle^M_h f(x) = \triangle^1_h(\triangle^{M-1}_h f)(x), \quad M = 2, 3, \ldots .$$

This definition also allows a direct formula

$$\triangle^M_h f(x) := \sum_{j=0}^{M} (-1)^j \binom{M}{j} f(x+(M-j)h).$$

By ball mean of differences we mean the quantity

$$d^M_t f(x) = t^{-n}\int_{|h|\le t} \left|\triangle^M_h f(x)\right| \mathrm{d}h = \int_B \left|\triangle^M_{th} f(x)\right| \mathrm{d}h,$$

where $B = \{y \in \mathbb{R}^n : |y| < 1\}$ is the unit ball of $\mathbb{R}^n$, $t > 0$ is a real number and M is a natural number.

Now we define the quasi-norms corresponding to generalized Besov Morrey spaces. Let $f \in \mathcal{M}^\varphi_q(\mathbb{R}^n)$. Then we define

$$\|f\|^*_{\mathcal{N}^s_{\mathcal{M}^\varphi_q,r}(\mathbb{R}^n)} \equiv \|f\|_{\mathcal{M}^\varphi_q} + \left(\int_0^\infty t^{-sr}\|d^M_t f\|^r_{\mathcal{M}^\varphi_q}\frac{\mathrm{d}t}{t}\right)^{1/r}.$$

Theorem 2.1. *Let $0 < q < \infty$, $0 < r \le \infty$, $s \in \mathbb{R}$ and $\varphi \in \mathcal{G}_q$. Furthermore, let $M \in \mathbb{N}$ with $M > s$. If*

$$s > \sigma_q \equiv n\left(\frac{1}{\min\{1,q\}} - 1\right),$$

then $\|f\|^*_{\mathcal{N}^s_{\mathcal{M}^\varphi_q,r}(\mathbb{R}^n)}$ and $\|f\|_{\mathcal{N}^s_{\mathcal{M}^\varphi_q,r}(\mathbb{R}^n)}$ are equivalent on $\mathcal{N}^s_{\mathcal{M}^\varphi_q,r}(\mathbb{R}^n)$.

References

[1] E. Nakai, *Hardy-Littlewood maximal operator, singular integral operators and the Riesz potentials on generalized Morrey spaces.* Math. Nachr. **166** (1994), 95–103.

[2] S. Nakamura, T. Noi and Y. Sawano, *Gereralized Morrey spaces and trace operator.* Sci. China Math. **59** (2016), 281–336.

Attractive points, acute points and approximation of fixed points of nonlinear mappings

厚芝幸子 (Sachiko Atsushiba)　(山梨大学大学院教育学研究科)*

Let H be a real Hilbert space with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$ and let C be a nonempty subset of H. For a mapping $T : C \to C$, we denote by $F(T)$ the set of *fixed points* of T and by $A(T)$ the set of *attractive points* [8] of T, i.e.,

(i) $F(T) = \{z \in C : Tz = z\}$;

(ii) $A(T) = \{z \in H : \|Tx - z\| \leq \|x - z\|, \ \ \forall x \in C\}$.

A mapping $T : C \to C$ is called *nonexpansive* if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. In 1975, Baillon [4] proved the following first nonlinear ergodic theorem in a Hilbert space: Let C be a nonempty bounded closed convex subset of a Hilbert space H and let T be a nonexpansive mapping of C into itself. Then, for any $x \in C$, $S_n x = \frac{1}{n} \sum_{i=0}^{n-1} T^i x$ converges weakly to a fixed point of T (see also [7]). Kocourek, Takahashi and Yao [5] introduced a broad class of nonlinear mappings called *generalized hybrid* which containing nonexpansive mappings, nonspreading mappings, and hybrid mappings in a Hilbert space. They proved a mean convergence theorem for generalized hybrid mappings which generalizes Baillon's nonlinear ergodic theorem. Motivated by Baillon [4], and Kocourek, Takahashi and Yao [5], Takahashi and Takeuchi [8] introduced the concept of attractive points of a nonlinear mapping in a Hilbert space and they proved a mean convergence theorem of Baillon's type without convexity for generalized hybrid mappings. Author and Takahashi [2] introduced the concept of common attractive points of a nonexpansive semigroup in a Hilbert space and proved a nonlinear mean convergence theorem of Baillon's type [4] without convexity for nonexpansive semigroups.

Motivated by Takahashi and Takeuchi [8], Atsushiba, Iemoto, Kubota and Takeuchi [3] introduce the concept of k-acute points. Let $k \in [0, 1]$. Let C be a subset of a Hilbert space H and T be a mapping of C into H. We define the set $\mathcal{A}_k(T)$ by

$$\mathcal{A}_k(T) = \{ \ v \in H : \ \|Tx - v\|^2 \leq \|x - v\|^2 + k\|x - Tx\|^2 \quad \text{for all} x \in C \ \}.$$

The author is supported by Grant-in-Aid for Scientific Research No. 26400196 from Japan Society for the Promotion of Science.

* Department of Mathematics, Graduate School of Education, University of Yamanashi, 4-4-37, Takeda, Kofu, Yamanashi 400-8510, Japan
e-mail: asachiko@yamanashi.ac.jp

We call $\mathcal{A}_k(T)$ the set of k-acute points of T. Because, in 2-dimensional Euclidean space setting, $\angle v\,x\,Tx$ is not an obtuse angle for $x \in C$ and $v \in \mathcal{A}(T)$. Further, we introduce the concept of common k-acute points of famlies of nonlinear mappings.

In this talk, we study the common k-acute points of families of nonlinear mappings. Then, we study some properties of common k-acute points and relations among k-acute points, attractive points and fixed points. We also study an iteration scheme for generalized hybrid mappings and prove convergence theorems. Further, we prove convergence theorems for some classes of nonlinear mappings.

参考文献

[1] S. Akashi and W. Takahashi, *Strong convergence theorem for nonexpansive mappings on star-shaped sets in Hilbert spaces*, Appl.Math. Comput. **219** (2012), 2035–2040.

[2] S. Atsushiba, W. Takahashi, *Nonlinear strong ergodic theorems for commutative nonexpansive semigroups on strictly convex Banach spaces*, J. Nonlinear Convex Anal. **1** (2013), 213–231.

[3] *Convergence theorems for some classes of nonlinear mappings in Hilbert spaces*, Linear and Nonlinear Anal. **2** (2016), 125–153.

[4] J.-B. Baillon, *Un theoreme de type ergodique pour les contractions non lineaires dans un espace de Hilbert*, C. R. Acad. Sei. Paris Ser. A-B **280** (1975), 1511 - 1514.

[5] P. Kocourek, W. Takahashi, and J.-C. Yao, *Fixed point theorems and weak convergence theorems for generalized hybrid mappings in Hilbert spaces*, Taiwanese J. Math. **14** (2010), 2497–2511.

[6] P. Kocourek, W. Takahashi, and J.-C. Yao, *Fixed point theorems and ergodic theorems for nonlinear mappings in Banach spaces*, Adv. Math. Econ. **15** (2011), 67–88.
Proc. Amer. Math. Soc. **96** (1986), 55-58.

[7] W. Takahashi, Nonlinear Functional Analysis, Yokohama Publishers, Yokohama, 2000.

[8] W. Takahashi and Y. Takeuchi, *Nonlinear ergodic theorem without convexity for generalized hybrid mappings in a Hilbert space*, J. Nonlinear Convex Anal. **12** (2011), 399–406.

[9] R. Wittmann, *Approximation of fixed points of nonexpansive mappings,* Arch. Math., **58** (1992) 486–491.

On direct sums of Banach spaces

加藤 幹雄 (Mikio Kato) 九州工大 名誉教授
田村 高幸 (Takayuki Tamura) 千葉大 人文社会

Direct sums of Banach spaces have been often treated in the context of geometric properties of Banach spaces as well as the fixed point property. We shall discuss three types of direct sums; ψ-, Z-, and A-direct sums. A ψ-direct sum is a Z-direct sum, which is an A-direct sum, while these notions are equivalent, that is, any A-direct sum is isometrically isomorphic the ψ-direct sum with some convex function ψ. Owing to this fact a sequence of previous results on ψ- and Z-direct sums will be generalized to the A-direct sum setting.

1. Definition Let $\|\cdot\|$ be a norm on $\mathbb{R}^N$.

(i) $\|\cdot\|$ is called *absolute* if $\|(z_1,\cdots,z_N)\| = \|(|z_1|,\cdots,|z_N|)\|$ for all $(z_j) \in \mathbb{R}^N$.

(ii) $\|\cdot\|$ is called *normalized* if $\|(1,0,\cdots,0)\| = \cdots = \|(0,\cdots,0,1)\| = 1$.

(iii) $\|\cdot\|$ is called *monotone* if $\big[|z_j| \le |w_j|\ (\forall j) \implies \|(z_j)\| \le \|(w_j)\|\big]$.

Let $AN_N :=$ {all absolute normalized (AN) norms on $\mathbb{R}^N$}.

2. Proposition (Bhatia) *A norm $\|\cdot\|$ on $\mathbb{R}^N$ is absolute if and only if it is monotone.*

3. Examples (i) ℓ_p-norms $(1 \le p \le \infty)$ are AN norms.

(ii) The following norm is neither absolute nor normalized: $\|(z_1,\ldots,z_N)\| = |z_1| + |z_1 - z_2| + \cdots + |z_1 - z_N|$.

4. Z-direct sum

Let $\|\cdot\|_Z$ be an arbitrary norm on $\mathbb{R}^N$ which is monotone in $\mathbb{R}^N_+$. In this case $\|\cdot\|_Z$ is called a *Z-norm* and one writes $Z = (\mathbb{R}^N, \|\cdot\|_Z)$.

The *Z-direct sum* $(X_1 \oplus \cdots \oplus X_N)_Z$ of $X_1,\ldots,X_N$ is their direct sum equipped with the norm

$$\|(x_1,\cdots,x_N)\|_Z := \|(\|x_1\|,\cdots,\|x_N\|)\|_Z \quad \text{for } (x_1,\ldots,x_N) \in X_1 \oplus \cdots \oplus X_N. \quad (1)$$

5. Remark on the definition of Z-direct sum

To construct a Z-direct sum the underlying Z-norm $\|\cdot\|_Z$ on $\mathbb{R}^N$ can be assumed to be absolute, which, in fact, is done in Dowling and Saejung [JMAA, **369** 2010]. In this case, since $\|\cdot\|_Z$ is aboslute if and only if $\|\cdot\|_Z$ is monotone, the monotonicity condition in $\mathbb{R}^N_+$ in the definition of Z-direct sum is superfluous and can be omitted. Therefore a Z-direct sum $(X_1 \oplus \cdots \oplus X_N)_Z$ may be conisdered as the direct sum constructed from an absolute norm $\|\cdot\|_Z$ on $\mathbb{R}^N$.

6. ψ-direct sum

A direct sum constructed from an absolute normalized norm $\|\cdot\|_{AN}$ on $\mathbb{R}^N$ as in (1) is called a *ψ-direct sum* and denoted by $(X_1 \oplus \cdots \oplus X_N)_\psi$, since for every absolute normalized norm $\|\cdot\|_{AN}$ there corresponds to a unique convex function ψ which is defined by

$$\psi(s) = \Big\|(1 - \sum_{i=1}^{N-1} s_i, s_1, \cdots, s_{N-1})\Big\|_{AN} \quad \text{for } s = (s_1, \cdots, s_{N-1}) \in \Delta_N,$$

where $\Delta_N = \{s = (s_1, \cdots, s_{N-1}) \in \mathbb{R}^{N-1} : \sum_{i=1}^{N-1} s_i \le 1,\ s_i \ge 0\}$. Therefore we write $\|\cdot\|_\psi$ for $\|\cdot\|_{AN}$ and refer to $\|\cdot\|_\psi$ as a *ψ-norm.*

7. ***A*-direct sum** (Dhompongsa-Kato-Tamura, LNA **1**, 2015)

Let $X_1, \dots, X_N$ be Banach spaces. Let $\|\cdot\|_A$ be an arbitrary norm on $\mathbb{R}^N$. The A-direct sum $(X_1 \oplus \cdots \oplus X_N)_A$ of $X_1, \dots, X_N$ is their direct sum equipped with the norm

$$\|(x_1, \dots, x_N)\|_A = \|(\|x_1\|, \dots, \|x_N\|)\|_A \quad \text{for } (x_1, \dots, x_N) \in X_1 \oplus \cdots \oplus X_N.$$

8. Theorem (Dhompongsa-Kato-Tamura, LNA **1**, 2015) *Let $\|\cdot\|_A$ be an arbitrary norm on $\mathbb{R}^N$. Then there exists $\psi \in \Psi_N$ such that $(X_1 \oplus \cdots \oplus X_N)_A$ is isometrically isomorphic to $(X_1 \oplus \cdots \oplus X_N)_\psi$. More precisely we have*

$$\|(x_1, \dots, x_N)\|_A = \|(a_1 x_1, \dots, a_N x_N)\|_\psi,$$

where $a_k = \|(0, \dots, 0, \overset{k}{\smile}{1}, 0, \dots, 0)\|_A$. In particular, any Z-direct sum is isometrically isomorphic to a ψ-direct sum.

9. Applications We shall apply Theorem 8 to some recent results on Z- and ψ-direct sums to obtain their general results.

References

[1] A. Betiuk-Pilarska and S. Prus, Uniform nonsquareness of direct sums Banach spaces, Topol. Methods Nonlinear Anal. **34** (2009), 181-186.

[2] S. Dhompongsa, M. Kato and T. Tamura, Uniform non-squareness for A-direct sums of Banach spaces with a strictly monotone norm, Linear Nonlinear Analysis **1** (2015), 247-260.

[3] P. N. Dowling and S. Saejung, Non-squareness and uniform non-squareness of Z-direct sums, J. Math. Anal. Appl. **369** (2010), 53-59.

[4] M. Kato, K.-S. Saito and T. Tamura, On the ψ-direct sums of Banach spaces and convexity, J. Aust. Math. Soc. **75** (2003), 413-422.

[5] M. Kato and T. Tamura, Weak nearly uniform smoothness of the ψ-direct sums $(X_1 \oplus \cdots \oplus X_N)_\psi$, Comment. Math. **52** (2012), 171-198.

[6] M. Kato and T. Tamura, On the uniform non-ℓ_1^n-ness and new classes of convex functions, J. Nonlinear Convex Anal. **16** (2015), 2225-2241.

[7] M. Kato and T. Tamura, On the uniform non-squareness of direct sums of Banach spaces, in preparation.

The Pérez inequality on weighted Morrey spaces

飯田毅士
(福島工業高等専門学校　一般教科)

重み付き Morrey 空間上の分数冪積分作用素に関する Pérez の不等式についての考察について述べる. はじめに, 分数冪積分作用素 I_α, 分数冪極大作用素 M_α を次のように定義する：

Definition 1. $0 < \alpha < n$ に対して

$$I_\alpha f(x) := \int_{\mathbb{R}^n} \frac{f(y)}{|x-y|^{n-\alpha}} dy.$$

$0 \le \alpha < n$ に対して

$$M_\alpha(f)(x) := \sup_{Q \ni x} |Q|^{\frac{\alpha}{n}} \fint_Q |f(y)| dy.$$

1995 年に, Pérez [1] は次の不等式を示した.

Theorem A. $1 < p < \infty$, $0 < \alpha < n$ とする.
(i) $\alpha p < n$ のとき,

$$\int_{\mathbb{R}^n} |I_\alpha f(x)|^p v(x) dx \le C \int_{\mathbb{R}^n} |f(x)|^p M_{\alpha p}\left(M^{[p]} v\right)(x) dx,$$

ここで, $M^{[p]} = M \circ M \circ \cdots \circ M$ とする.
(ii) $1 < p < \infty$ のとき,

$$\int_{\mathbb{R}^n} |I_\alpha f(x)|^p v(x) dx \le C \int_{\mathbb{R}^n} M_\alpha f(x)^p M^{[p]+1} v(x) dx.$$

特に, $p = 1$ のとき,

$$\int_{\mathbb{R}^n} |I_\alpha f(x)| v(x) dx \le C \int_{\mathbb{R}^n} M_\alpha f(x) M v(x) dx.$$

ここで, $[\cdot]$ はガウス記号とする. すなわち $[p]$ は p を超えない最大の整数とする.

Remark 1. Theorem A の (i), (ii) の右辺において, $[p]$ を $[p]-1$ に置き換えることが出来ないことを Pérez [1] が示した.

すなわち, (i) において, $[p]$ を $[p]-1$ とした次の不等式は一般には成り立たない.

$$\int_{\mathbb{R}^n} |I_\alpha f(x)|^p\, v(x)dx \leq C \int_{\mathbb{R}^n} |f(x)|^p\, M_{\alpha p}\left(M^{[p]-1}v\right)(x)dx.$$

実際, $f = v = \chi_B$, $B = B(0,1)$ を原点を中心とする単位円とすると,

$$\int_{\mathbb{R}^n} |I_\alpha f(x)|^p\, v(x)dx = \infty, \int_{\mathbb{R}^n} |f(x)|^p\, M_{\alpha p}\left(M^{[p]-1}v\right)(x)dx < \infty$$

となる.

(ii) において, $[p]+1$ を $[p]$ に置き換えた次の不等式は一般には, 成り立たない.

$$\int_{\mathbb{R}^n} |I_\alpha f(x)|^p\, v(x)dx \leq C \int_{\mathbb{R}^n} M_\alpha f(x)^p M^{[p]}v(x)dx.$$

実際, $n = 1$, $1 < p < 2$, $f(y) = \chi_{(0,1)}(y)|y|^{-\alpha}$, さらに任意の $0 < \varepsilon \leq p-1$ に対して,

$$v(x) = \chi_{\left(0,\frac{1}{e}\right)}(x)\frac{1}{x\left(\log\frac{1}{x}\right)^{\varepsilon+2}}$$

とおくと,

$$\int_{\mathbb{R}^n} |I_\alpha f(x)|^p\, v(x)dx = \infty, \int_{\mathbb{R}^n} M_\alpha f(x)^p M^{[p]}v(x)dx < \infty$$

となる.

参考文献

[1] C. Pérez, *Sharp L^p-weighted Sobolev inequalities*, Ann. Inst. Fourier (Grenoble), 45, (1995), 809-824.

The Fefferman-Stein type inequality for the directional maximal operator

齋藤 洋樹 (Academic support center, Kogakuin University)*1
田中 仁 (National University Corporation Tsukuba University of Technology)*2

Introduction and Results

Let $\mathcal{T}: L^p(\mathbb{R}^n) \to L^p(\mathbb{R}^n)$, $p > 1$, be a sublinear operator. It is a fundamental problem of the weight theory that to determine some maximal operator $\mathcal{M}_{\mathcal{T}}$ capturing certain geometric characteristics of $\mathcal{T}$ such that

$$\int_{\mathbb{R}^n} |\mathcal{T}f(x)|^p w(x)\,dx \le C \int_{\mathbb{R}^n} |f(x)|^p \mathcal{M}_{\mathcal{T}} w(x)\,dx, \tag{1}$$

where w is an arbitrary weight, i.e., non-negative measurable function on $\mathbb{R}^n$. Under such circumstances, a duality argument allows (1) to transfer bounds on $\mathcal{M}_{\mathcal{T}}$ to bounds on $\mathcal{T}$. Thus given such an operator $\mathcal{T}$ and an index p, it is worthy to identify a corresponding geometrically defined maximal operator $\mathcal{M}_{\mathcal{T}}$ satisfying (1).

For the Hardy-Littlewood maximal operator $\mathcal{M}$, it is well known that

$$\int_{\mathbb{R}^n} \mathcal{M}f(x)^p w(x)\,dx \le C \int_{\mathbb{R}^n} |f(x)|^p \mathcal{M}w(x)\,dx$$

holds for an arbitrary weight w and $p > 1$, and further that

$$\sup_{t>0} t\, w(\{x \in \mathbb{R}^n : \mathcal{M}f(x) > t\}) \le C \int_{\mathbb{R}^n} |f(x)|\, \mathcal{M}w(x)\,dx, \tag{2}$$

where we write for $E \subset \mathbb{R}^n$ $w(E) = \int_E w(x)\,dx$. These are called the Fefferman-Stein inequality and are toy models of the problem. (see [1].)

The purpose of this talk is to establish the Fefferman-Stein type inequality for the directional maximal operator. To state our results precisely, we will describe some definitions. Let $0 < p < \infty$ and w be a weight. We define the weighted Lebesgue space $L^p(\mathbb{R}^n, w)$ to be a Banach space equipped with the usual norm defined by the usual manner.

Let Σ be a set of unit vectors in $\mathbb{R}^2$, i.e., a subset of the unit circle S^1. Associated with Σ, we define $\mathcal{B}_\Sigma$ to be the set of all closed rectangles in $\mathbb{R}^2$ whose longest side is parallel to some vector in Σ. We also define the directional maximal operator $\mathfrak{M}_\Sigma$ associated with Σ as

$$\mathfrak{M}_\Sigma f(x) = \sup_{R \in \mathcal{B}_\Sigma} \mathbf{1}_R(x) \fint_R |f(y)|\,dy.$$

The first author is supported by Grant-in-Aid for Young Scientists (B) (15K17551), the Japan Society for the Promotion of Science. The second author is supported by Grant-in-Aid for Scientific Research (C) (15K04918), the Japan Society for the Promotion of Science.
2010 Mathematics Subject Classification: 42B25
キーワード:directional maximal operator; Fefferman-Stein type inequality; Hardy-Littlewood maximal operator
*1 2665–1, Nakanomachi, Hachioji-shi Tokyo, 192–0015, Japan
e-mail: j1107703@gmail.com
*2 e-mail: htanaka@k.tsukuba-tech.ac.jp

Many authors studied this operator and showed that $\mathfrak{M}_\Sigma$ is bounded on $L^2(\mathbb{R}^2)$ with the operator norm $O(\log N)$ for any set Σ with cardinality N. For fixed sufficiently large $N \in \mathbb{Z}^+$, let Σ_N be the set of N uniformly spread directions on the circle S^1. The following result is the main result of this talk which is a weighted version of the classical result due to J.-O. Strömberg [9].

Theorem 1 *Let $N > 10$ and w be any weight on $\mathbb{R}^2$. Let $W = \mathfrak{M}_{\Sigma_N}\mathcal{M}w$. There exists a constant $C > 0$ such that for all $f \in L^2(\mathbb{R}^2, W)$ we have*

$$\sup_{t>0} t\, w(\{x \in \mathbb{R}^2 : \mathfrak{M}_{\Sigma_N} f(x) > t\})^{1/2} \leq C(\log N)^{1/2}\|f\|_{L^2(\mathbb{R}^2,W)}. \tag{3}$$

By Marcinkiewicz's interpolation between (3) and the trivial $L^\infty(\mathbb{R}^2, W) \to L^\infty(\mathbb{R}^2, w)$ estimate, we have the following corollary:

Corollary 0.1 *Let $N > 10$ and w be any weight on $\mathbb{R}^2$. Let $W = \mathfrak{M}_{\Sigma_N}\mathcal{M}w$. For $2 \leq p < \infty$, there exists a constant $C_p > 0$ such that for all $f \in L^p(\mathbb{R}^2, W)$ we have*

$$\|\mathfrak{M}_{\Sigma_N} f\|_{L^p(\mathbb{R}^2,w)} \leq C_p(\log N)^{1/p}\|f\|_{L^p(\mathbb{R}^2,W)}. \tag{4}$$

When $N = 2$, the operator $\mathfrak{M}_{\Sigma_N}$ corresponds the well-known strong maximal operator $\mathfrak{M}_S$. Theorem 1 is valid for *any* weight and the proof in the next section can be simply modified for the case $N = 2$, to provide the following result. See also [2, p472].

Corollary 0.2 *Let w be any weight on $\mathbb{R}^2$ and let $W = \mathfrak{M}_S\mathcal{M}w$. There exists a constant $C > 0$ such that for all $f \in L^2(\mathbb{R}^2, W)$ we have*

$$\sup_{t>0} t\, w(\{x \in \mathbb{R}^2 : \mathfrak{M}_S f(x) > t\})^{1/2} \leq C\|f\|_{L^2(\mathbb{R}^2,W)}. \tag{5}$$

At the end of this talk, we will discuss about the case $1 < p < 2$.

参考文献

[1] C. Fefferman and E. M. Stein *Some Maximal Inequalities*, Amer. J. math., **93** (1971), no. 1, 107–115.

[2] J. Garcia-Cuerva and J. L. Rubio de Francia, *Weighted Norm Inequalities and Related Topics*, North-Holland, Math. Stud., **116** (1985).

[3] L. Grafakos, *Modern Fourier Analysis*, volume 250 of Graduate Texts in Mathematics, Springer, New York, 2nd edition, 2008.

[4] B. Jessen, J. Marcinkiewicz and A. Zygmund *Note on the differentiability of multiple integrals*, Fund. Math. **25** (1935), 217–234.

[5] N. H. Katz, *Maximal operators over arbitrary sets of directions*, Duke Math. J., **97** (1999), no. 1, 67–79.

[6] N. H. Katz, *Remarks on maximal operators over arbitrary sets of directions*, Bull. London Math. Soc., **31** (1999), no. 6, 700–710.

[7] T. Luque and I. Parissis *The endpoint Fefferman-Stein inequality for the strong maximal function*, J. Funct. Anal. **266** (2014), no. 1, 199–212.

[8] T. Mitsis, *The weighted weak type inequality for the strong maximal function*, J. Fourier Anal. Appl. **12** (2006), no. 6, 645–652.

[9] J. O. Strömberg, *Maximal functions associated to rectangles with uniformly distributed directions*, Ann. Math. (2), **107** (1978), no. 2, 399–402.

Lorentz improvement of the Blascamp-Lieb inequality

Yoshihiro Sawano (Tokyo Metropolitan University)

The Blascamp-Lieb inequality unifies the Hölder inequality, the Young inequality and the Loomis-Whitney inequality. Chirst has pointed out that a refinement via the Lorentz index is ok in some special cases. Christ gave a sufficient condition for such cases. Here we point out that this condition is necessary. If we consider the general case, we need more conditions for the improved Blascamp-Lieb inequality to hold. We just point out that we actually need it. A general necessary and sufficient condition is still open. This is a joint work with Dr. Shohei Nakamura, Professor Neal Bez and Professor Sanghyuk Lee.

Pointwise multipliers on Musielak-Orlicz spaces

中井 英一（茨城大学理学部）
Eiichi NAKAI (Ibaraki University)

Let (Ω, μ) be a complete σ-finite measure space. We denote by $L^0(\Omega)$ the set of all measurable functions from Ω to $\mathbb{R}$ or $\mathbb{C}$. Let E_1 and E_2 be subspaces of $L^0(\Omega)$. We say that a function $g \in L^0(\Omega)$ is a pointwise multiplier from E_1 to E_2, if the pointwise multiplication fg is in E_2 for any $f \in E_1$. We denote by $\mathrm{PWM}(E_1, E_2)$ the set of all pointwise multipliers from E_1 to E_2. We abbreviate $\mathrm{PWM}(E, E)$ to $\mathrm{PWM}(E)$.

For $p \in (0, \infty]$, we denote by $L^p(\Omega)$ the usual Lebesgue spaces. It is well known as Hölder's inequality that

$$\|fg\|_{L^{p_2}(\Omega)} \le \|f\|_{L^{p_1}(\Omega)} \|g\|_{L^{p_3}(\Omega)},$$

for $1/p_2 = 1/p_1 + 1/p_3$ with $p_i \in (0, \infty]$, $i = 1, 2, 3$. This shows that

$$\mathrm{PWM}(L^{p_1}(\Omega), L^{p_2}(\Omega)) \supset L^{p_3}(\Omega).$$

Conversely, we can show the reverse inclusion by using the closed graph theorem or the uniform boundedness theorem. That is,

$$\mathrm{PWM}(L^{p_1}(\Omega), L^{p_2}(\Omega)) = L^{p_3}(\Omega).$$

Moreover, the operator norm of $g \in \mathrm{PWM}(L^{p_1}(\Omega), L^{p_2}(\Omega))$ is equal to $\|g\|_{L^{p_3}(\Omega)}$.

In this talk we extend the above equality to Musielak-Orlicz spaces. For example we can show the following: Let p_i be variable exponents, w_i be

---

実解析学シンポジウム 2016 アブストラクト
The author was supported by Grant-in-Aid for Scientific Research (B), No. 15H03621, Japan Society for the Promotion of Science.

weight functions, $i = 1, 2, 3$, and

$$\Omega_\infty = \{x \in \Omega : p_3(x) = \infty\}.$$

Assume that $\inf_{x\in\Omega} p_i(x) > 0$, $i = 1, 2, 3$, and $\sup_{x\in\Omega\setminus\Omega_\infty} p_3(x) < \infty$.

Example 1. Let

$$\frac{1}{p_1(x)} + \frac{1}{p_3(x)} = \frac{1}{p_2(x)}.$$

Then

$$\mathrm{PWM}(L^{p_1(\cdot)}(\Omega), L^{p_2(\cdot)}(\Omega)) = L^{p_3(\cdot)}(\Omega),$$

and

$$\mathrm{PWM}(\exp(L^{p_1(\cdot)})(\Omega), \exp(L^{p_2(\cdot)})(\Omega)) = \exp(L^{p_3(\cdot)})(\Omega).$$

Example 2. Let

$$\frac{1}{p_1(x)} + \frac{1}{p_3(x)} = \frac{1}{p_2(x)}, \quad w_1(x)^{1/p_1(x)} w_3(x)^{1/p_3(x)} = w_2(x)^{1/p_2(x)}.$$

Then

$$\mathrm{PWM}(L^{p_1(\cdot)}_{w_1}(\Omega), L^{p_2(\cdot)}_{w_2}(\Omega)) = L^{p_3(\cdot)}_{w_3}(\Omega).$$

References

[1] L. Maligranda and E. Nakai, *Pointwise multipliers of Orlicz spaces*, Arch. Math. 95 (2010), 251–256.

[2] L. Maligranda and L. E. Persson, *Generalized duality of some Banach function spaces*, Indag. Math. 51 (1989), no. 3, 323–338.

[3] E. Nakai, *Pointwise multipliers*, Memoirs of The Akashi College of Technology, 37 (1995), 85–94.

[4] E. Nakai, *Pointwise multipliers on Musielak-Orlicz spaces*, Nihonkai Math. J., to appear.

[5] H. Nakano, Modulared semi-ordered linear spaces, Maruzen Co., Ltd., Tokyo, 1950.

Eiichi Nakai, email address: eiichi.nakai.math@vc.ibaraki.ac.jp

Maximal functions associated with non-isotropic dilations of hypersurfaces in R^3

Wenjuan Li (Northwest Polytechnical University)

The goal of this article is to establish L^p-estimates for maximal functions associated with nonisotropic dilations of hypersurfaces in $\mathbb{R}^3$.

Several results have already been obtained by Greenleaf, Iosevich-Sawyer, Ikromov-Kempe-Müller and Zimmermann, but for some situations such as the hypersurface parameterized as the graph of a smooth function $\Phi(x_1, x_2) = x_2^d(1 + \mathcal{O}(x_2^m))$ near the origin, where $d \geq 2$, $m \geq 1$, and associated dilations $\delta_t(x) = (t^a x_1, tx_2, t^d x_3)$ for an arbitrary real number $a > 0$, the question was open until recently. In fact, such problems do arise already in lower dimensions. For instance, we consider the curve $\gamma(x) = (x, x^2(1 + \phi(x)))$ and associated dilations $\delta_t(x) = (tx_1, t^2 x_2)$. If $\phi \equiv 0$, then the corresponding maximal function is the maximal function along parabolas in the plane, which plays an important role in the study of singular Randon transforms, and which is very well understood due to the work by Nagel-Riviere-Wainger and others. If $\phi \neq 0$ and $\phi(x) = \mathcal{O}(x^m)$, $m \geq 1$, the problem was open until recently, however, the corresponding maximal function shows features related to the Bourgain circular maximal function, which required deep ideas and local smoothing estimates established by Mockenhaupt-Seeger-Sogge for Fourier integral operators satisfying the so-called "cinematic curvature" condition. However, we observe that in the study of $\mathcal{M}$ related to the mentioned curve $\gamma(x)$ and associated dilations, we will consider a family of corresponding Fourier integral operators which fail to satisfy the "cinematic curvature condition" uniformly, which means that classical local smoothing estimates could not be directly applied to our problem. In this talk, we will introduced some new ideas in order to overcome the above difficulty and finally establish sharp L^p-estimates for the maximal function related to the curve $\gamma(x)$ with associated dilations in the plane. Later, we generalize the result to curves of finite type d ($d \geq 2$) and associated dilations $\delta_t(x) = (tx_1, t^d x_2)$. Furthermore, we also obtain L^p-estimates for the maximal function related to the mentioned hypersurface $\Phi(x_1, x_2)$ in $\mathbb{R}^3$ with associated dilations. Moreover, by an alternative approach, we also get L^p-estimates for some particular classes of maximal functions in $\mathbb{R}^3$ established earlier by Greenleaf, Iosevich-Sawyer, Ikromov-Kempe-Müller and Zimmermann.

Characterization of Two Weight Norm Inequality for Littlewood-Paley g_λ^*-Function

Qingying Xue (Beijing Normal University)

Let $n \geq 2$ and g_λ^* be the well-known Littlewood-Paley function of higher dimension associated with Poisson kernel, which was defined and studied by E. M. Stein.In this talk, we will give a characterization of two weight norm inequality for g_λ^*-function. We showed that:

$$\left\|g_\lambda^*(f\sigma)\right\|_{L^2(w)} \lesssim \left\|f\right\|_{L^2(\sigma)}$$

if and only if the two weight Muchenhoupt A_2 condition holds, and a testing condition holds, where (w, σ) is a pair of weights. We actually proved this characterization for g_λ^* function associated with more general fractional Poisson kernel $p^\alpha(x) = (1 + |x|^2)^{-(n+\alpha)/2}$. Moreover, the corresponding results for intrinsic g_λ^*-function were also presented.